

Directional Separable Kernel Family with Compact Support -SKCS- for de-noising image and preserving edges

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Abstract— Due to the noisy sensor or the channel transmission errors, images are often corrupted by noise. Linear smoothing filters remove high-frequency components such as Gaussian filter. These isotropic kernels are very efficient to remove noise but, it blurred the edges of the image. In terms of edges preservation and noise reduction using Gaussian filter, several form are proposed such as the kernel family with compact supports (KCS) and its separable version family namely SKCS. This new kernel has also a compact support and preserves the most important properties of the Gaussian kernel in order to perform image de-nosing. In this paper, an anisotropic and directional Gaussian separable kernel is introduced in order to improve the SKCS efficiency. A practical comparison is established between results obtained by using the conventional and the directional proposed SKCS operators. Extensive simulations indicate that the proposed method perform significantly better in terms of noise suppression and detail preservation than the classic one.

Keywords— Image filtering, Kernel with Compact Support, Separable Kernel version, Anisotropic Gaussian filter, orientation, directional.

I. INTRODUCTION

The most important problem in image processing is image de-nosing. Gaussian kernel is an efficient low pass filtering method for removing noises but it has some inconvenient such as the blurred of the edges and the borders. These inconvenient are related to the static shape of its kernel. To overcome this problem of the traditional Gaussian filter, many techniques and methods have been introduced and proposed in order to improve the quality of corrupted images by noise and protect edges. Scale-space representation has been developed by the computer vision community, in particular by Witkin [1] and Koenderink [2]. In their work, Remaki and Cheriet [3] proposed a new kernel family with compact supports (KCS) in scale space derived from the Gaussian. The main idea of scale-space representation is to embed the original image in a one-parameter family of derived images, in which the fine scale details are successively suppressed [4, 5 and 9]. It consists of using a convolution with a chosen kernel in the smoothing operation. [6, 7 and 8] introduce a separable version of the KCS kernel. The new approach define an implementation based on the

decomposition of the Laplacian of the KCS shape, this kernel is able to recover the information loss while reducing processing time. In this work we are interested in low-pass filtering for de-noising purpose based on directional separable version of the KCS. The selection of the parameters changes according to gradient magnitudes. We have shown that the proposed separable, anisotropic and directional kernels are able to eliminate only high frequencies related to the noise, and protect the main details. Our comparison is based on edges conservation and improvement of the signal to noise ratio. This paper is organized as follows: In section 2, we review some properties and limitations of Gaussian and KCS kernel. Then, we present the separable version SKCS and we explain the principle of our proposed approach. In the next section, we present the experimental results of the improved version of the SKCS. Finally, a conclusion and remarks are given.

II. PRESENTATION OF THE GAUSSIAN KERNELS AND IT'S DERIVATIVE

A. Gaussian kernel

A Gaussian blur (also known as Gaussian smoothing) is the result of blurring an image by a Gaussian function as shown by the following equation :

$$G(x, y, \mu, \sigma) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(x-\mu_1)^2 - (y-\mu_2)^2}{2\sigma^2}\right] \quad (1)$$

Where σ^2 is the variance which determines the width of the Gaussian distribution and μ is the mean (location of the peak).

The distribution with $\mu = 0$ and $\sigma^2 = 1$ is called the standard normal. A plot of the 1D Gaussian functions with different values of the mean is shown in Figure 1:

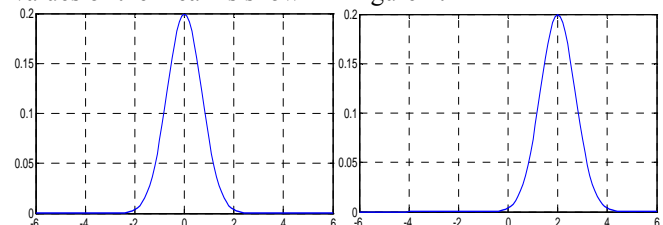


Fig.1 1D profile of the Gaussian kernel

Several authors show that the Gaussian kernel is unique and offers many beneficial properties such the separate, Gaussian functions are rotationally symmetric, its description by the diffusion equation. The size of Gaussian filter is limited to the window $[-\infty, +\infty]$. This infinite support and must be truncated to a finite window. The surface covered by the curve within the range $[-3\sigma, +3\sigma]$ is equal to 99.74 %, while the range $[-4\sigma, +4\sigma]$ covers 99.99 %. thus if we aim to cover near to 100 % of the information to be processed, then the mask size of smoothing filter should be within the range $[-6\sigma, +6\sigma]$ [10]. To overcome the problem of the blur of the smoothed image using Gaussian kernel, Different multi-scale representations have been proposed. A new kernel family with compact supports in scale space that achieved good performance in extracting data information with regard to the Gaussian kernel [3,4 and 9].

B. KCS Kernel

Unfortunately, the Gaussian kernel has two practical limitations: information loss caused by the unavoidable Gaussian truncation and the prohibitive processing time due to the mask size. To recover the information loss when using the Gaussian kernel while they drastically reduce the processing time, a new kernel with compact support (KCS) is created. It gives an explicit relation between the “classical” scale parameter, namely the standard deviation and the mask size of the kernel concretely: mask size = 2σ (σ is the standard deviation). Using a topological transform of the plan of the R^2 space to a unit ball, this kernel is represented by the following expression:

$$\rho_{\sigma,\gamma}(x,y) = \begin{cases} \frac{1}{C_\gamma \sigma^2} e^{\gamma \left(\frac{\sigma^2}{x^2+y^2-\sigma^2} + 1 \right)} & \text{If } x^2 + y^2 < \sigma^2 \\ 0 & \text{elsewhere} \end{cases} \quad (2)$$

Where

C_γ : is the normalization constant

σ : is the standard deviation KCS

γ : is the width parameter.

The parameter γ controls the distance between the zero-crossing of the KCS to the origin of the axis. The support of ρ_{σ} is $B(0, \sigma)$.

This kernel keeps the most important properties of the Gaussian kernel, but obviously not all those that make the Gaussian kernel unique. These properties are respectively: i) recovering the initial signal (image) when the scale parameter tends to zero (a necessary condition to build a scale-space); ii) continuity with respect to scale parameter; iii) the strong regularization property; and iv) the zero crossing diminishing property [5 and 9]. Figure 2 depicts 2D profiles of the KCS kernel:

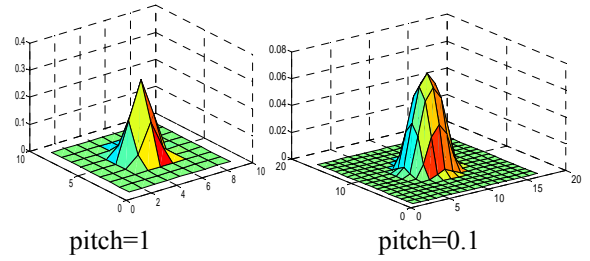


Fig.2 two-dimensional profiles of the KCS Kernel

In scale -space formulation, a convolution product with the kernel is computed at each scale. These successive convolutions require substantial processing time. Considerable savings of time may be achieved by using separable convolution masks. For this purpose, a Separable version of the KCS, denoted SKCS (Separable Kernel with Compact Support) was presented in order to increase the KCS performances.

C. Separable version of the KCS Kernel

The KCS formula is shown in (2). As we can see, this formula is not separable. However, the product of 1D KCS following x and y , leads to a new 2D operator [6]. To design the new proposed kernel, we define an implementation based on the decomposition of the Laplacian of the KCS shape, as the sum of two 1D filters. It is defined by the following formula:

$$SKCS(x,y) = KCS(x) \times KCS(y) \quad (3)$$

The generalized SKCS formula:

$$\phi_{\sigma,\gamma}(x,y) = \begin{cases} \lambda^2 \cdot e^{\left(\frac{\gamma\sigma^2}{x^2-\sigma^2}\right)} \cdot e^{\left(\frac{\gamma\sigma^2}{y^2-\sigma^2}\right)} & \text{if } \begin{cases} x^2 < \sigma^2 \\ y^2 < \sigma^2 \end{cases} \\ 0 & \text{elsewhere} \end{cases} \quad (4)$$

Where $\lambda = \frac{e^\gamma}{C_\gamma \sigma}$ where $C_\gamma = \int_{-1}^1 e^{\left(\frac{\gamma}{x^2-1}\right)} dx$ is the normalization constant.

In order to investigate the SCKS on de-noising images, extracting the important information and edges from noisy and degraded image, we try to vary the core of the filter. To do so, we introduce the mean parameter to the previous formula, as follows:

$$\phi_{\sigma,\gamma}(x,y) = \begin{cases} \lambda^2 \cdot e^{\left(\frac{\gamma\sigma^2}{(x-\mu_1)^2-\sigma^2}\right)} \cdot e^{\left(\frac{\gamma\sigma^2}{(y-\mu_2)^2-\sigma^2}\right)} & \text{if } \begin{cases} (x-\mu_1)^2 < \sigma^2 \\ (y-\mu_2)^2 < \sigma^2 \end{cases} \\ 0 & \text{elsewhere} \end{cases} \quad (5)$$

The development of the algorithm based on edge preservation and improvement of the filtered image's quality is summarized as follow:

Step1: edge detection.

Step2: Filter the noisy image based on a decision computed from 8 neighborhoods comparison:

- If $I(x,y)$ is an edge compute the difference between this pixel and its 8 neighboring $N(x,y)$ (equation 6), in this

case, The values of the mean (μ_1 and μ_2) are determined according to the maximum variation (gradient).

• Else, filter the selected window using a 3×3 static SKCS where ($\mu_1 = \mu_2 = 0$).

$$N(x,y) = \begin{cases} I(x,y) - I(x-1,y-1) \\ I(x,y) - I(x-1,y) \\ I(x,y) - I(x-1,y+1) \\ I(x,y) - I(x,y-1) \\ I(x,y) - I(x,y+1) \\ I(x,y) - I(x+1,y-1) \\ I(x,y) - I(x+1,y) \\ I(x,y) - I(x+1,y+1) \end{cases} \quad (6)$$

For 1 to 8 (number of neighbors of each pixel), if $N(x,y)$ is the higher variation, μ_1 and μ_2 are determined automatically to define the position of the higher ponderation of the separable KCS version. Near the contour, the image shows a strong gradient. It is therefore preferable to encourage the diffusion in the direction parallel to the latter and not in the direction of the gradient, to better preserve the discontinuity. Figure 3 a model in a local set (ξ, η) whose basis vectors are respectively normal and tangential of the contour.

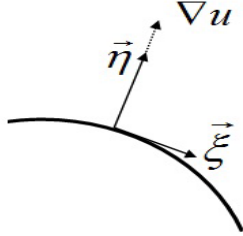


Fig.3 Image contour in local set (ξ, η)

The conventional method, for the orientation is given by the following equations:

$$\theta = \arg(\vec{\nabla}f) = \tan^{-1} \left[\frac{G_y}{G_x} \right] \quad (7)$$

Where

$$\vec{\nabla}f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \quad (8)$$

$$G_x = \sum_{h=-b}^b \sum_{k=-b}^b (\nabla_x(x+h, y+k))^2 \quad (9)$$

$$G_y = \sum_{h=-b}^b \sum_{k=-b}^b (\nabla_y(x+h, y+h))^2 \quad (10)$$

The magnitude of the gradient is given by:

$$\left| \vec{\nabla}f \right| = \sqrt{G_x^2 + G_y^2} \quad (11)$$

The next figure shows the orientation of the skcs kernel in which the gradient is orthogonal to the edges.

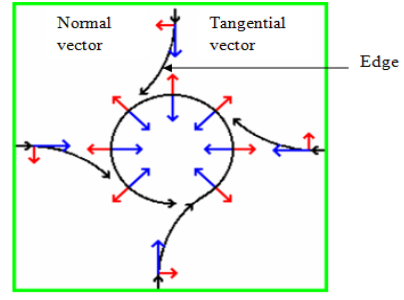


Fig.4 Gradient orientation

In order to evaluate the performance of the proposed method, we first apply it on binary image de-noising, than to demonstrate its efficiency, we smooth some gray-scale images.

III. EXPERIMENTAL RESULTS

We perform our experiments on some images in order to show the improvement of the separable version of the skcs Kernel Family with Compact Support. The difference between the efficiency of the static and the directional SKCS version is based on computing of the signal to noise ratio (PSNR, equation 12) and the correlation between the original and the enhanced image.

$$PSNR = 10 \cdot \log_{10} \left(\frac{d^2}{MSE} \right) \quad (12)$$

$$\text{Where } MSE = \frac{1}{M \times N} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \left([f(x,y) - r(x,y)]^2 \right) \quad (13)$$

Experimental results show that the proposed technique provides very important performance.

The next figure shows an image which is corrupted with salt and pepper noise with an intensity of 0.01:

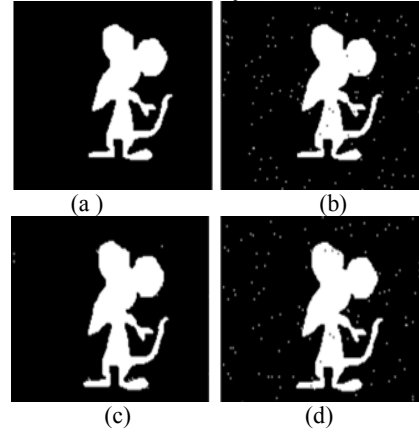


Fig.5. smoothing results: (a) original image, (b) noisy image, (c) directional skcs filtered image, (d) conventional skcs filtered image

Applying a zoom option on the edges, we show that with the directional skcs filter, the borders and the details are preserved but with the conventional skcs, the noise persists:



Fig6. Directional and conventional SKCS filter

The following table proves that using the proposed SKCS filter with compact support, generates better results due the values of the PSNR and the correlation are important than the static one.

TABLE I COMPARISON BETWEEN THE PROPOSED METHOD, THE CONVENTIONAL SKCS FILTER

Noise density	0.01	0.02	0.03	0.04	0.05
Directional PSNR	62.03	59.6	59.45	59.3	57.35
Static PSNR	54.83	52.51	50.66	49.65	48.67
Δ PSNR	7.2	7.082	8.78	9.65	8.682
Directional correlation	0.996	0.992	0.986	0.98	0.975
Static correlation	0.978	0.958	0.938	0.92	0.901
Δ correlation	0.018	0.034	0.048	0.06	0.074

Where

$$\Delta PSNR = PSNR_{\text{based directional kernel}} - PSNR_{\text{Static}} \quad (14)$$

It's clear that the proposed technique is more efficient than the conventional one because of the important difference between the psnr results.

In the next experiment, the density of noise is 0.04, the result of different technique filtering methods :

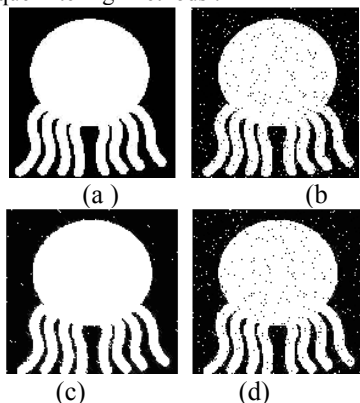


Fig.7. Filtered images

For the image on figure 8, we applied different type of noise and we compute the psnr and the correlation. It's clear that the improved version of the SKCS give better results of filtered image:



Fig.8. original image

TABLE II DIFFERENT PSNR'S VALUES

Noise	PSNR values		
	Proposed technique	traditional technique	Δ PSNR
Gaussian	50.47	49.36	1.108
localvar	54.57	46.61	7.955
Salt&Pepper	55.28	52.72	2.55

The previous results show that the proposed technique is more efficient than the conventional method; this performance is shown due to:

- The Good quality of the filtered image.
- The borders and edge are preserved.

In the next experiment, we apply our method on gray-scale images. Applying the proposed technique to de-noise 'cameraman' image from Gaussian noise, we illustrated these results of PSNR:

TABLE III. FILTERING TECHNIQUE 'S PSNR

Gaussian noise	0.01	0.02	0.03	0.04
Directional PSNR	45.19	46.33	48.77	49.47
Static PSNR	37.63	38.378	39.08	39.62
Δ PSNR	7.565	7.957	9.6846	9.85

The proposed noise reduction technique is efficient than the traditional filtering method do the value of PSNR. Also, this efficiency occurs without removing the details such as with classical filter. Figure 9 show that with the dynamic SKCS filter, edges and borders are preserved and noise is reduced.

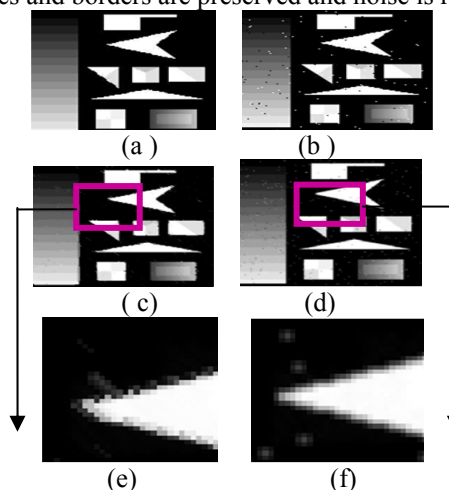


Fig.9. Comparison between the two filtering technique,(a) original image ,(b) noisy image,(c) directional SKCS filtering,(d) traditional SKCS filtering

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The results of PSNR results while we increase the variance of the Gaussian noise are represented in the next table:

TABLE.IV. PSNR VARIATION

Noise density	PSNR values		
	Proposed technique	traditional technique	Delta
0.01	51.55	50.72	0.8318
0.02	50.9	47.85	3.05
0.03	50.65	45.98	4.673
0.04	50.5	44.57	5.928
0.05	50.15	43.32	6.83
0.06	49.7	42.94	6.77

IV. CONCLUSION

In this paper, we have proposed a new image de-noising approach based on edge detection and gradient decision. The experimental results are very promising in terms of reducing noise while preserving edges and borders. It should be mentioned that in this paper we tested our proposed technique on de-noising binary and gray-scale images with different intensity and type of noise. PSNR and correlation results show that the directional SKCS filter gives better results than the conventional filter.

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